Week on Stochastic Dynamics

Programa Temático - Dynamics Beyond Uniform Hyperbolicity
IMPA, Rio de Janeiro, de 7/10 até 11/10.

Abstracts

Monday, 7

Singular Stochastic Dynamics

Salah-Eldin A. Mohammed

Southern Illinois

Abstract:
We show the existence of a unique stochastic flow of Sobolev diffeomorphisms for stochastic differential equations (SDE's) with bounded measurable drift coefficients. This result is counter-intuitive: The dominant `culture' in stochastic (and deterministic) dynamical systems is that the flow `inherits' its spatial regularity from the driving vector fields. Spatial regularity of the stochastic flow yields existence and uniqueness of a Sobolev differentiable weak solution of the (Stratonovich) stochastic transport equation with singular coefficients (cf. work by Kunita (1990); and Flandoli-Gubinelli-Priola (2010)).
The corresponding deterministic transport equation does not in general have a solution (Ambrosio (2004)). If time permits, we construct a Sobolev differentiable stochastic flow of diffeomorphisms for one-dimensional SDEs driven by bounded measurable diffusion coefficients. No uniqueness of solutions to the SDE is presumed! This is joint work with T. Nillsen and F. Proske (Oslo, Norway).

Weak functional stochastic calculus and local-time representations

Alberto Ohashi

UFPB

Abstract:
In this talk, we establish a weak version of functional stochastic calculus w.r.t to weak-Dirichlet-type processes which extends the previous works of Dupire and Fournie and Cont. The main novelty of our approach is a Sobolev- type differential structure which allows us to represent a large class of Wiener functionals in terms of weak derivative operators and space-time local time integrals interpreted in the pathwise Young sense. Applications to hedging in stochastic volatility models and backward SDEs will be discussed.

Tuesday, 8

Geometric Currents associated to diffusions

Diego Ledesma

UNICAMP

Abstract:
We extend the idea of asymtotic cycles of dynamical systems to the case of L-diffusions on a manifold M. In particular, for each $L-$diffusion we define a closed current $\mathcal{J}_L:\Omega^1(M)\rightarrow \mathbb{R}$ that is associated to homological cycles of M. We will do application to Brownian motion with drifts and to the Foliated Brownian Motion.

Exponential growth rate for a singular linear stochastic delay differential equation

Michael Scheutzow

TU, Berlin

Abstract:
First we briefly review sufficient criteria for the uniqueness of an invariant measure of a stochastic delay differential equation obtained in joint work with Martin Hairer and Jonathan Mattingly.
Then, we study the very simple one-dimensional equation $dX(t)=X(t-1)dW(t)$ in more detail and establish the existence of a deterministic exponential growth rate of a suitable norm of the solution via a Furstenberg-Hasminskii-type formula.

Wednesday, 9

Stochastic Calculus on Vector Bundles

Pedro Catuogno

UNICAMP

Abstract:
The aim of this talk is to relate covariant stochastic integration in a vector bundle E with the usual Stratonovich calculus via the connector $\K:TE \rightarrow E$ which carries the connection dependence.

Weak functional Itô calculus and applications to non-markovian optimal stopping problems

Dorival Leão Jr

ICMC-USP

Abstract:
In this talk, we establish a weak version of functional stochastic calculus w.r.t to weak-Dirichlet-type processes. Our approach is based on weak approximations developed by Leão and Ohashi [2013] and extends the pathwise functional Itô calculus developed by Dupire [2009], Cont and Fournie [2013] and the Sobolev-type differential structure introduced by Peng and Song [2013]. As an illustration of the theory, we show a novel characterization of optimal stopping times by means ofdiscrete-type variational inequalities connected to fully non-Markovian systems in continuous-time.

Thursday, 10

Regularization by noise of Linear and Semilinear PDEs

Christian Olivera [1]
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Abstract

We show that linear and semilinear PDEs by stochastic perturbation are well-posedness meanwhile uniqueness may fail for the deterministic PDEs

An averaging principle for diffusions in foliated manifolds

Paulo Ruffino

UNICAMP

Abstract:
Consider an SDE on a foliated manifold whose trajectories lay on compact leaves. We investigate the effective behaviour of a small transversal perturbation of order ϵ. An average principle is shown to hold such that the component transversal to the leaves converges to the solution of a deterministic ODE, according to the average of the perturbing vector field with respect to invariant measures on the leaves, as ϵ goes to zero. An estimate of the rate of convergence is given. These results generalize the geometrical scope of previous approaches, including completely integrable stochastic Hamiltonian system.

Degenerate semigroups and stochastic flows of mappings in foliated manifolds

P. H. da Costa
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Abstract

Let $(M, \mathcal{F})$ be a compact Riemannian foliated manifold. We consider a family of compatible Feller semigroups in $C(M^n)$ associated to laws of the n-point motion. Under some assumptions (Le Jan and Raimond) there exists a stochastic flow of measurable mappings in M. We study the degeneracy of these semigroups such that the flow of mappings is foliated, i.e. each trajectory lays in a single leaf of the foliation a.s, hence creating a geometrical obstruction for coalescence of trajectories in different leaves. As an application, an averaging principle is proved for a first order perturbation transversal to the leaves. Estimates for the rate of convergence are calculated.

Key words: Feller semigroup, n-point motion, foliated space, stochastic flow of mappings, averaging principle.

Friday, 11

Ruelle's theorem in a random setting - methods of proof and new phenomena

Manuel Stadlbauer

UFBA

Abstract:
In this talk we give a survey of the methods of proof available in case of a random environment. Since Ruelle's operator is a random Markov operator, these methods also have implications in the theory of classical random processes in random environment. Finally, a basic application of Ruelle's theorem to random fractals is given. These examples reveal new phenomena arising from the randomization.

Stochastic n-point bifurcation of Lévy flows

Michael Högele

Potsdam, Alemanha

Abstract:
In his famous book on stochastic flows Kunita proves a remarkable result for Brownian flows. It states that the uniqueness of a flow preserving measure can be characterized by the shape of the invariant measures of the one-point and the two-point process of the flow. This property is of high importance in order to detect bifurcations of the flow. We prove that for Lévy flows this property is in general not satisfied. After a short introduction to Lévy flows we present the construction of the simplest example in a sense, where this fails. This is joint work with Paulo Ruffino (Unicamp) and Pedro Catuogno (Unicamp).

Long-term behavior of locally linearized integrators for stochastic oscillators

Hugo de La Cruz

FGV

Abstract:
We propose numerical schemes for the long-term integration of stochastic differential equations with additive noise. By focusing on a linear stochastic oscillator as a test equation, it is shown that these methods are able to reproduce key features related to the long time behavior of this system: the linear growth of the second moment of the solution, an infinitely-oscillation property and the symplectic structure of this system. Computer experiments illustrate the theoretical findings and the advantages over long periods of time of the proposed method in comparison with some conventional integrators.